Imperfect Competition in Online Auctions*

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Abstract

We study online auctions, where two sellers sequentially chooseers2

Sealed-bid auctions were prevalent before the advent of the Internet, but have lost their popularity due to a drastic improvement in the communication technologies and reduction of search costs for buyers.

One of the important attributes of e-commerce is the ease of trading. Previously, companies had to incur xed costs to set up at least one distribution channel. Reselling those items after the purchase was also problematic due to high search and coordination costs. These days, any individual may almost costlessly bring a product to an online consumer-to-consumer (C-2-C) market, whether for the purposes of resale or as a uniquely crafted item. The latter tendency produces a distinctive environment in which there may be onlrk(searc)28(i41.53ion)- Td [()27(7(1n)-d-g)-393ionmaltuue homogeneusl-322 rhuslehold-2385(tems)051 from-5385()nsaions and-3385(hern-4385(actions)-43 low enough to safeguard against being undercut by the second seller, and in equilibrium the first-arriving seller makes more profit than the second-arriving seller. In addition to characterizing an equilibrium in the above environment, this note has a methodological contribution. We show how the approach of Myerson (1981) can be extended to a case with two sellers receiving some split of the expected revenue generated from the buyers.

Peters and Severinov (2006) prove that when there are many sellers and buyers in online-auction markets, the reserve prices set by the sellers are equal to their marginal costs. In contrast to sealed-bid auctions characterized by simultaneous choice of reserve prices, it is unlikely that in online markets sellers choose reserve prices simultaneously. Rather, a seller who comes to the market first, chooses a reserve price expecting a subsequent arrival of another seller. In principle, sellers may have a good estimate of how many competitors to anticipate. Such a strategic environment may be framed as a Stackelberg-like model where sellers choose reserve prices, and our results are consistent with the standard symmetric Stackelberg model, in which the first-moving seller has an advantage and earns a higher profit.

Our note is related to Burguet and SÆkovics (1999), who show that the results of McAfee (1993) and Peters and Severinov (1997) hold only for large markets where many sellers offer sealed-bid auctions. The crucial feature of the environment considered by this literature is the commitment of buyers, who could no longer switch to another auction after placing a bid in one of them. Burguet and SÆkovics (1999) argue that in a duopoly the reserve prices are no longer driven to marginal costs. The authors consider simultaneous choice of reserve prices by the sellers and find that the equilibrium exists only in mixed strategies. When the choice of reserve prices is sequential (which reflects the observed regularities of online markets), we show that there is a unique equilibrium outcome. Due to differences in the behavior of buyers faced with either sealed-bid or ascending auctions, Burguet and SÆkovics (1999) could not use the marginal revenue approach (Myerson (1981), Bulow and Roberts (1989), Bulow and Klemperer (1994)), which is applicable in our analysis and allows us to tremendously simplify calculations further generalize our results to any selling mechanisms in which only the highest valued buyers are awarded units.

We show that just like in the environment considered by Burguet and SÆkovics (1999), competition between two sellers competing in online auctions is not enough to drive reserve prices to marginal costs. To our knowledge, there is no empirical literature examining the structure of reserve prices in online auction markets. Our theory predicts variation to exist even with two sellers. This contrasts with a monopolist who sells items by auctions at the same optimal reserve price and a competitive market in which reserve prices are equal to marginal costs. The monopolist outcome may also arise if competing duopolists were to collude. Hence, the absence of variation in the reserve prices on particular segments of C-2-C markets could potentially be used as a test for collusion.

In the next section we describe the model. In section 3 we describe the sellers' profits directly and then adapt the revenue equivalence theorem to rewrite the sellers' profits. In section 4 we describe the equilibrium. Section 5 provides an example with three buyers with uniformly distributed values and shows how the reserve price of the first-moving seller is just high enough to discourage the second-arriving seller from undercutting. We conclude in section 6 by considering online auctions (in which buyers can bid simultaneously in both auctions), but in which the sellers choose reserve prices simultaneously to better understand the role of sequentially chosen reserve prices.

2 The model

There are two sellers with identical costs (normalized to zero) a and b each possessing a single unit. There are n buyers, each demanding a single unit. The values of the buyers are i.i.d, drawn from distribution $F(\cdot)$ with support $[0, \bar{v}]$; $F(\cdot)$ is differentiable with everywhere positive density $f(\cdot)$. Let the vector of valuations be $v = (v_1, v_2, \ldots, v_n)$ and the vector of sorted values be $x = (x_1, x_2, \ldots, x_n)$ with $x_1 \geq x_2 \geq \ldots \geq x_n$. In other words, the elements of x are order statistics. Let $f_k(x)$ denote the marginal density function of the k^{th} highest order statistic and let $f^{(n)}_{1:k}(x_1, x_2, \ldots, x_k)$

selects reserve price r_b. Each buyer submits a sealed bid. The allocation is according to the seller-offer double auction, which works as follows. Make a single list, sorting the reserve prices and bids from highest to lowest, with ties ordered randomly. Set price P equal to the reserve price or bid in the n^{th} lowest position on this list. All sellers amongst the n^{th} lowest positions will sell a unit and receive P dollars; all buyers with values in the remaining 2 highest positions will purchase a unit and pay P dollars. The remaining sellers and buyers do not transact. This means that the price paid by

3 Sellers' profits

In the seller-offer double auction or in decentralized ascending price auctions, only the buyers with the highest valuations win units, as described in the prior section. We next give seller profit functions based on whether the seller has the lower or higher reserve price. Name the reserve prices such that $r_2 \geq r_1$. We first treat the case when $r_2 > r_1$. The seller with reserve price r_2 sells a unit at price r_2 if $x_1 \geq x_2 \geq r_2 > x_3$ and at price x_3 if $x_1 \geq x_2 \geq x_3 \geq r_2$ for expected profit of:

$$\begin{aligned}\pi_2(r_2) = & \int_{x_3=0}^{x_3=r_2}\int_{x_2=r_2}^{x_2=\bar{v}}\int_{x_1=x_2}^{x_1=\bar{v}} r_2 f_{1:3}^{(n)}(x_1,x_2,x_3)\,dx_1dx_2dx_3 + \\ & \int_{x_3=r_2}^{x_3=\bar{v}}\int_{x_2=x_3}^{x_2=\bar{v}}\int_{x_1=x_2}^{x_1=\bar{v}} x_3 f_{1:3}^{(n)}(x_1,x_2,x_3)\,dx_1dx_2dx_3. \end{aligned} \quad (1)$$

The seller with reserve price r_1 sells a unit at price r_2 if $x_1 \geq x_2 \geq r_2 > x_3$ and at price x_3 if $x_1 \geq x_2 \geq x_3 \geq r_2$ as before, and also at price r_1 if $x_1 \geq r_1 > x_2$ and at price x_2 if $r_2 \geq x_2 \geq r_1$ for the expected profit of:

$$\begin{aligned}\pi_1(r_1,r_2) = & \pi_2(r_2) + \int_{x_2=0}^{x_2=r_1}\int_{x_1=r_1}^{x_1=\bar{v}} r_1 f_{1:2}^{(n)}(x_1,x_2)\,dx_1dx_2 + \\ & \int_{x_2=r_1}^{x_2=r_2}\int_{x_1=x_2}^{x_1=\bar{v}} x_2 f_{1:2}^{(n)}(x_1,x_2)\,dx_1dx_2. \end{aligned} \quad (2)$$

Despite the particulars of the payments, the revenue equivalence theorem (Myerson (1981), Riley and Samuelson (1981), Krishna (2009)) indicates that what matters is the allocation of units: in a single-unit demand independent private values setting (as in our model), in any incentive compatible mechanism in which a buyer with value 0 gets an expected payoff of 0, the expected revenue equals the expected marginal revenue of the buyers awarded units, where marginal revenue is defined as $MR(z) := z - \frac{1-F(z)}{f(z)}$. We may thus express the profit functions as summarized in the following proposition.

Proposition 1. An equivalent way to express the pro t functions is:

$$_2(r_2) = \frac{1}{2}\int_{x_2=r_2}^{x_2=\bar{v}}\int_{x_1=x_2}^{x_1=\bar{v}} MR(x_1) + MR(x_2)\ \ f^{(n)}_{1:2}$$

Assumption 1. The marginal revenue function $MR(z) := z - \frac{1-F(z)}{f(z)}$ is regular: that is, it is continuous and strictly increasing.

We will also make use of the following quick result introduced in Bulow and Roberts (1989), which can be shown using integration by parts:

Lemma 1. For all p with $0 \leq p \leq \bar{v}$, we have:

$$\int_{p}^{\bar{v}} MR(z) f(z) dz = p[1 - F(p)]:$$

The next three lemmas establish important properties of the profit functions.

Lemma 2. For all $(r_1; r_2)$ with $0 < r_1 \leq r_2 < \bar{v}$, we have:

$$\Pi_1(r_1; r_2) > \Pi_2(r_2):$$

Proof. This follows immediately from equations (1) and (2). □

Lemma 3. For all r with $0 < r < \bar{v}$, we have:

$$\Pi_1(r; r) > \Pi_0(r):$$

Proof. By Lemma 2, $\Pi_1(r; r) > \Pi_2(r)$. By definition, Π_0 is a convex combination of $\Pi_1(r; r)$ and $\Pi_2(r)$ and thus lies somewhere in between: $\Pi_1(r; r) > \Pi_0(r) > \Pi_2(r)$. □

Lemma 4. The function $\Pi_2(\cdot)$ defined on $[0; \bar{v}]$ is single-peaked, and reaches its peak at $r_2^* := \phi^{-1}(0)$, where $\phi(r_2) := r_2 + MR(r_2)$. Each function in the family $\{\Pi_1(\cdot\,; r_2)\}_{r_2 \in [0;\bar{v}]}$, with $\Pi_1(\cdot\,; r_2)$ defined on $[0; r_2]$, is single-peaked and reaches its peak at $\min\{r_1^*; r_2\}$, where $r_1^* := MR^{-1}(0)$.

Proof. Use Proposition 1 to get:

$$\begin{aligned}
\frac{d\Pi_2(r_2)}{dr_2} &= \frac{1}{2}\int_{x_1=r_2}^{x_1=\bar{v}} \left[MR(x_1) + MR(r_2)\right] n(n-1) f(x_1) f(r_2) F^{n-2}(r_2) dx_1 \\
&= \frac{1}{2} n(n-1) f(r_2) F^{n-2}(r_2) \left[(1-F(r_2)) r_2 + MR(r_2)(1-F(r_2))\right] \\
&= \frac{1}{2} \underbrace{n(n-1) f(r_2) F^{n-2}(r_2)(1-F(r_2))}_{f_2(r_2) \geq 0} (r_2 + MR(r_2))
\end{aligned}$$

where Lemma 1 gives the second equality. This derivative equals 0 when r_2 is 0 or $\bar{v}$, but otherwise takes sign opposite of $(r_2) := r_2 + MR(r_2)$. Because $(0) < 0 < (\bar{v})$ and (r_2) is continuous and strictly increasing by Assumption 1 (regularity), there is a unique value of r_2 in the interior of $[0; \bar{v}]$ with $(r_2) = 0$: $r_2 = {}^{1}(0)$. Thus, in the interior of $[0; \bar{v}]$, $d\ _2(r_2)/dr_2$ begins positive, equals zero at r_2, and turns negative, thereby giving the single-peakedness of $_2(r_2)$.

Next, use Proposition 1 to get:

$$
\begin{aligned}
\frac{@\ _1(r_1; r_2)}{@r_1} &= \int_{x_2=0}^{x_2=r_1} MR(r_1)n(n\quad 1)f(r_1)f(x_2)F^{n\quad 2}(x_2)dx_2 \\
&= MR(r_1)nf(r_1)\int_{x_2=0}^{x_2=r_1}(n\quad 1)f(x_2)F^{n\quad 2}(x_2)dx_2 \\
&= \underbrace{nf(r_1)F^{n\quad 1}(r_1)}_{f_1(r_1)\ \ 0} MR(r_1):
\end{aligned}
$$

A similar argument to the prior paragraph gives the single-peakedness of $_1(r_1; r_2)$ at $r_1 = MR\ ^{1}(0)$ whenever $r_1\quad r_2$ and otherwise at r_2, noting that $_1(\ ; r_2)$ is only de ned on $[0; r_2]$. □

Lemma 5. The following ranking holds: $0 < r_2 < r_1$.

Proof. The function $(r_2) = r_2 + MR(r_2)$ is strictly increasing and continuous by Assumption 1. By de nition, $MR(r_1) = 0$ and $(r_2) = 0$. The result follows from $(0) = MR(0) = \quad 1/f(0) < 0$ and $(r_1) = r_1 + MR(r_1) = r_1 > 0$

a strategy for each player, such that after every history, the payo to a player whose move it is cannot be improved by this player unilaterally deviating to another strategy.

Consider the value $\bar{r}_1$ such that $_1(\bar{r}_1; \bar{r}_1) = \ _2(r_2)$. Note that $_1(r_2; r_2) > \ _2(r_2)$ by Lemma 2 and $_1(0; 0) = \ _2(0) < \ _2(r_2)$ by Lemma 4. Note also that $_1(r; r)$ is strictly increasing in r for all $r \ 2 \ [0; r_2]$. This follows because for $0 \quad r < s \quad r_2$, we have $_1(r; r) < \ _1(r; s) < \ _1(s; s)$, where the rst inequality comes from equation (2), and the fact that $_2(r) < \ _2(s)$ comes from the single-peakedness result of Lemma 4, price r_a. We examine the best response of seller b. If $r_a = \bar{v}$, seller b has a unique best response to price r_1, using Lemma 4 and noting that choosing reserve price $\bar{v}$ results in zero pro t. If $r_a = 0$, observe that

$$
{}_0(0) = \frac{1}{2}\ {}_1(0;0) + \frac{1}{2}\ {}_2(0) = \ {}_2(0) < \ {}_2(r_2)
$$

where the inequality is from Lemma 4. Thus, choosing reserve price r_2 is better than matching with a reserve price of 0, and is therefore the unique best response. For the remaining cases of r_a, we may appeal to Lemma 3 to note that matching this reserve price is never a best response for seller b. From Lemma 4 it follows that seller b does best whenever he chooses a lower reserve price to get as close as possible to r_1 and does best whenever he chooses a higher reserve price to get as close as possible to r_2.

Case 1: $r_1 < r_a < \bar{v}$. Seller b can achieve ${}_1(r_1; r_a)$ by pricing below r_a

Proposition 3 shows that reserve prices are not driven down to the sellers' marginal costs, resulting in inefficiency. In addition, seller a who moves first sets a lower price and earns a higher profit than seller b since $\pi_1(\bar{r}_1; r_2) > \pi_2(r_2)$ by Lemma 2. As a remark, it follows from the aforementioned revenue equivalence theorem that if sellers were to collude to maximize their joint profits, they would set both reserve prices at $MR^{-1}(0) = r_1 > r_2$. Thus, in a non-cooperative game with sellers moving sequentially, the equilibrium results in more social surplus (including the buyers) than in a monopolized or cartelized market.

5 Numerical example

Suppose that there are $n = 3$ buyers, with values distributed (uniformly) on $[0; 1]$. Then, $F(v) = v$, $f(v) = 1$, and marginal revenue is $MR(z) = 2z - 1$. Using Proposition 1, the profit functions for sellers with the higher and lower reserve prices are:

$$\pi_2(r_2) = \frac{1}{2}\int_{x_2=r_2}^{x_2=1}\int_{x_1=x_2}^{x_1=1}(2x_1 - 1 + 2x_2 - 1)6x_2\,dx_1dx_2 = \frac{9}{4}r_2^4 - 4r_2^3 + \frac{3}{2}r_2^2 + \frac{1}{4}$$

and

$$\pi_1(r_1; r_2) = \pi_2(r_2) + \int_{x_2=0}^{x_2=r_1}\int_{x_1=r_1}^{x_1=1}(2x_1 - 1)6x_2\,dx_1dx_2 +$$

$$\int_{x_2=r_1}^{x_2=r_2}\int_{x_1=x_2}^{x_1=1}(2x_1 - 1)6x_2\,dx_1dx_2 = -\frac{3}{2}r_1^4 + r_1^3 + \frac{3}{4}r_2^4 - 2r_2^3 + \frac{3}{2}r_2^2 + \frac{1}{4}$$

noting that the joint density of the two highest order statistics is $f_{1:2}^{(3)} = 6x_2$.

We obtain $r_2 = 1/3 \approx 0.333$ by solving $r_2 + MR(r_2) = 0$ or $r_2 + 2r_2 - 1 = 0$. This is the value of r_2 that maximizes (2

6 Conclusion

In this note we analyzed imperfect competition in online markets where two sellers enter the market sequentially and list their items by ascending auctions. We showed that the equilibrium outcome is unique with the first-arriving seller setting a low reserve price, and the second seller setting a higher reserve price. The first-moving seller receives larger expected profit, which is consistent with the first-mover advantage of the Stackelberg model. The equilibrium outcome is inefficient, because both reserve prices are set higher than the sellers' marginal costs.

Two more factors drive the results. The first one is the ability of buyers to switch costlessly between auctions, which is a likely feature of online markets. In the extreme case, buyers may procure bots scanning for desired goods across different digital auction platforms and bidding on their behalf. This behavior leads to well-structured profit functions for the sellers. The second factor is that sellers have identical costs. Many sellers in online consumer-to-consumer markets are reselling previously bought items, so one would not expect significant variability in the costs of sellers.

To conclude, we briefly consider the case of online auctions but where the two sellers simultaneously choose their prices. This case differs only from Burguet and Sákovics (1999) in the assumption that buyers can freely buy from either seller rather than commit to one seller's auction or the other's. It is straightforward to show using our profit functions (and similar to our proof of Lemma 2) that no pure strategy Nash equilibrium exists. Furthermore, it can be shown that the support of any mixed strategy equilibrium must be identical but cannot include zero, the same result as Burguet and Sákovics (1999) obtain when buyers commit to one auction or the other. Further, using a standard argument, the support of prices that a seller mixes over in any mixed strategy equilibrium must not contain any gaps or atoms. It can be shown that the supremum of the support of each seller's mixed strategy equals r_2 the price the second-moving seller chooses in the equilibrium in the version of the game when sellers choose reserve prices sequentially. If it were lower, then any seller pricing near enough this supremum would be the high priced seller with probability near enough one that it 662(Man)27(y0t81(that)-39 [redacted] close to) $_2(r_2)$

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