1. Let $P = 1!2!3!!!n = n!$ and let $S = 1 + 2 + 3 + É + n$, where n is a positive integer.

   (a) Prove that if n is odd, then S divides P exactly.

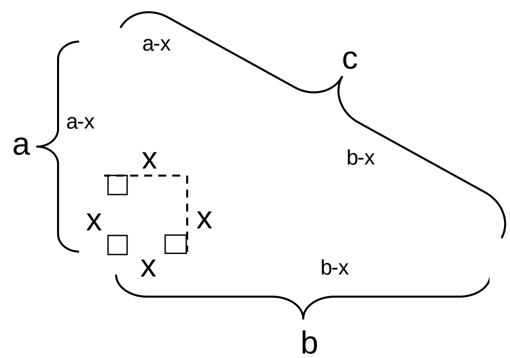
   (b) Prove that the converse of part (a) is <u>not</u> true.

2. Let $_{1}\ _{2}\ _{3}\ _{7}$

1. (a) If n is odd, then $n = 2k + 1$ where k is a positive integer. Then

$$\frac{P}{S} = \frac{1!\,2!\,3!!!\,(2k+1)}{1+2+3+\ldots+(2k+1)} = \frac{(2k+1)!}{\frac{1}{2}(2k+1)(2k+2)} = \frac{(2k+1)!}{(2k+1)(k+1)}$$

For every <u>odd</u> integer k, $ak^2 + bk$ is the sum of two odd integers, and therefore $ak^2 + bk$ is an even integer. So f(k) = $ak \quad + bk) + c$



Next consider either of the remaining two circles

□