THE 2016–2017 KENNESAW STATE UNIVERSITY
HIGH SCHOOL MATHEMATICS COMPETITION

PART I – MULTIPLE CHOICE

[illegible]

NO CALCULATORS

90 MINUTES

1. The average of ten numbers is 190. If one of the numbers is doubled, the new average is 500. What is the value of the number that was doubled?

   (A) 310 (B) 380 (C) 690 (D) 3100 (E) 3800

2. If the circumference of a circle is decreased by 20%, by what percent is the area of the circle decreased?

   (A) 20 (B) 32 (C) 36 (D) 40 (E) 64

3. Three students took a six-question true-false exam. Debbie answered #1 and #2 true and the rest false, Don answered #2 and #3 true and the rest false, and Chuck answered #3 and #4 true and the rest false. If Debbie and Don each got five of the questions correct, what is the largest possible number of correct answers that Chuck could have?

   (A) 1 (B) 2 (C) 3 (D) 4 (E) 5

4. Mary wanted to complete the 4x4 grid shown by filling in the individual squares so that each row and each column contained each of the numbers 1, 2, 3, 4 exactly once. What is the value of x?

			1
	2		
		x	
1			4

   (A) 1 (B) 2 (C) 3 (D) 4 (E) It cannot be done

5. The nine digits, 1, 2, 3, 4, 5, 6, 7, 8, 9 are written in a straight line in a certain order so that:
   (i) if any two digits add up to 10 they are written adjacent to each other, and
   (ii) if any two digits add up to 9 they are written adjacent to each other.

   The second digit from the left is odd. What is the seventh digit from the left?

   (A) 2 (B) 3 (C) 4 (D) 6 (E) 8

6. A square is plotted on a coordinate axis system. When the four x-coordinates and the four y-coordinates of the square's vertices are added, the sum is 348. If the coordinates of the center of the square are $(x, 37)$, compute the value of x.

(A) 30 (B) 35 (C) 40 (D) 45 (E) 50

7. Let $f(x) = ax^7 + bx^3 + cx - 5$. If $f(-7) = 7$, what is the value of $f(7)$?

(A) -17 (B) -12 (C) -7 (D) 12 (E) 17

8. A palindrome is a number which reads the same forwards and backwards (for example 383 and 15051 are palindromes).

13. How many values of x, $0° \le x < 360°$, satisfy the equation $\sec x + \tan x + \cos x = 0$?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

14. Dr. Garner asked his math students to find the sum of all three terms of a geometric sequence in which the middle number was missing. Unfortunately, one student confused geometric sequences with arithmetic sequences, and then completed the problem with no other mistakes, obtaining an answer of 351. If all the numbers used by Dr. Garner and the student were distinct positive integers, and if the ratio of the geometric sequence was an integer, what was the correct answer to Dr. Garner's problem?

(A) 248 (B) 279 (C) 294 (D) 343 (E) 387

15. The sum of the 3-digit numbers $\underline{3}\,\underline{5}\,\underline{x}$ and $\underline{4}\,\underline{y}\,\underline{7}$ is divisible by 36. Compute the smallest possible sum $x + y$.

(A) 6 (B) 8 (C) 9 (D) 10 (E) 12

16. Let $N = (k + 1) + (k + 2) + \ldots + (k + 31)$, where k is a positive integer. Compute the sum of the two smallest values of k for which N is a perfect square.

(A) 62 (B) 108 (C) 123 (D) 141 (E) None of these

17. Given two regular polygons with m and n sides, $m > n$, such that the degree measure of each interior angle of each polygon is a multiple of 7. All the vertices of the n-sided polygon are vertices of the m-sided polygon. Compute $m + n$.

(A) 25 (B) 39 (C) 53 (D) 60 (E) 81

18. The three-digit base b number $\underline{A}\,\underline{B}\,\underline{C}_{\,b}$ equals 169_{10}. If A, B, and C are distinct, and $A \ne 0$, determine the number of possible positive integral bases b.

(A) 3 (B) 4 (C) 5 (D) 6 (E) 7

19. Given the polynomial $f(x) = ax^3 + 2016x^2 + c$, with a and c nonzero. If c is a root of $f(x) = 0$, which of the following must represent the sum of the remaining two roots?

(A) $\dfrac{a}{c}$ (B) $\dfrac{-2016}{c}$ (C) $\dfrac{1}{ac}$ (D) $\dfrac{2016}{ac}$ (E) $\dfrac{ac}{2016}$

20. For all possible values of a fixed constant a, the system $x + ay = 2016$ and $ax + y = 2016a + 3$ has exactly three integral pairs of solutions, (x_1, y_1), (x_2, y_2), and (x_3, y_3). Compute the value of $x_1y_1 + x_2y_2 + x_3y_3$.

(A) 1008 (B) 2016 (C) 3024 (D) 4032 (E) None of these

21 In triangle ABC, AB = 7, BC = 19, and AC = 24. A circle of radius 5 is constructed with center B, intersecting $\overline{AC}$ at points D and D#. Compute the measure of $DBD#.

(A) 30¡ (B) 45¡ (C) 60¡ (D) 75¡ (E) 90¡

22. The expansions of $(1 + x)^n$ and $(1 + x)^{n+1}$ are each written in <u>ascending</u> powers of x. The ratio of the fourth term of the expansion of $(1 + x)^n$ to the fifth term of the expansion of $(1 + x)^{n+1}$ is $\frac{2}{3x}$. If n % 3, what is the value of n?

(A) 5 (B) 6 (C) 7 (D) 8 (E) 9

23. Let S be the sum of the base 10 logarithms of all of the proper divisors of 1,000,000. (By proper divisor of a natural number we mean a positive integral divisor other than 1 and the number itself.) What is the integer nearest to S?

(A) 100 (B) 123 (C) 141 (D) 147 (E) 150

24. A square is constructed so that its sides are parallel to the coordinate axes. When lines with slopes $\frac{1}{3}$ and $\frac{2}{3}$ are drawn through the center of the square, it is divided into two congruent pentagons and two congruent triangles. Compute the ratio of the area of one of the pentagons to the area of one of the triangles.

(A) $\frac{8}{1}$ (B) $\frac{9}{1}$ (C) $\frac{10}{1}$ (D) $\frac{11}{1}$ (E) $\frac{12}{1}$

25. The standard deviation, &, of a set of N numbers is a measure of “spread” from the mean, μ. It is given by the formula $! = \sqrt{\frac{\#(x_i \text{ " } \mu)^2}{N}}$, where x_i represent the individual numbers in the set (i= 1,2,3,…, N). A statistics student taking a test on standard deviation made an unfortunate error. Copying a set of 20 numbers, he accidentally left off one of the numbers, a 28. Thus he was only aware of 19 numbers. As a result, his value of

!

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<u>Solutions</u>:

1. Let the ten original numbers be represented as $a_1, a_2, a_3, \ldots, a_{10}$. Then
$\frac{a_1 + a_2 + a_3 + \ldots + a_{10}}{10} = 190$ and $a_1 + a_2 + a_3 + \ldots + a_{10} = 10(190) = 1900$.
Similarly, $2a_1 + a_2 + a_3 + \ldots + a_{10} = 10(500) = 5000$. Subtracting the last two equations, we obtain $a_1 = 3100$.

2. Since the circumference of a circle is directly proportional to the radius, the radius is also decreased by 20%. Therefore, the new area is $\pi\left(\frac{4}{5}r\right)^2 = \frac{64}{100}\pi r^2$. Thus the area is decreased by 36%.

3. Since Debbie and Don each got five of the questions correct, there are two possibilities.
   (i) Questions 1, 2, and 3 are true, and questions 4, 5, 6 are false.
   (ii) Question 2 is true and questions 1, 3, 4, 5, 6 are false.

   In (i) Chuck has three correct answers (questions 3, 5, and 6). In (ii) Chuck also has three correct answers, (questions 1, 5, 6). Either way, the most possible correct for Chuck is 3.

4. First fill in the bottom row, then the second column, then the top row, then the third cChuck

8. B $a + \frac{1}{a} = \frac{a^2+1}{a}$. Since $32^2 = 1024$, which is more than three digits, $10 \le a \le 31$. The answers can now be obtained by inspection: $10 + \frac{1}{10} = \frac{101}{10}$ and $25 + \frac{1}{25} = \frac{626}{25}$ and no others will work. Therefore, $a = 10$ or 25 and the desired sum is 35.

9. D Construct radius $\overline{PQ}$ and segment $\overline{QA}$. Then QA = PA = PQ = 1.

Therefore, $\triangle APQ$ is equilateral with side length 1. The length of the altitude of the triangle is $\frac{\sqrt{3}}{2}$. Since $\overline{AB}$ is twice the length of the altitude, $AB = \sqrt{3} \approx 1.732$.

10. E One of the three digits must be a 5. At least one of the remaining two digits must be even. If both are even, there are ${}_4C_2 = 6$ choices. If one is even and the other odd, there are ${}_4C_1 \cdot {}_4C_1 = 16$ choices. Therefore, there are 22 numbers divisible by 10. Since there are ${}_9C_3 = 84$ ways of choosing three digits, the desired probability is $\frac{22}{84} = \frac{11}{42}$.

11. E Clearly, $x = 1$ is a solution. The given equation can be rewritten $x^{x\sqrt{x}} = x^{\left(\frac{3}{2}x\right)}$. Thus, if $x \neq 1$, then $x\sqrt{x} = \frac{3}{2}x$. Squaring both sides and clearing fractions, $4x^3 - 9x^2 = 0$. Factoring, we obtain $x^2(4x - 9) = 0 \Rightarrow x = \frac{9}{4}$. The desired sum is $\frac{13}{4} = 3.25$

16. C (k + 1) + (k + 2) + É + (k + 31) = 31k + (1 + 2 + É + 31) = 31k + ________