1. Let , , and *A* all

From case 1, we know that $\cos \quad = \frac{!\ 4}{2 \quad !\ 2}$.

Using the law of cosines on BMC, $\mathrm{BM}^2 \quad \left(\frac{n}{2}\right)^2 \quad (n-1)^2 - 2\left(\frac{n}{2}\right)(n-1)\cos C$,

Substituting $C = \frac{n\ !}{n\ !}$, we obtain

$$\mathrm{BM}^2 = \left(\frac{}{2}\right)^2 + (\quad -1)^2 - 2\left(\frac{}{2}\right)(\quad -1)\left(\frac{-4}{2 \quad -2}\right) \text{ from which } \mathrm{BM}^2 = \frac{3n^2 + 4}{4}$$

If BM = $\frac{n}{}$, then $\left(\frac{n}{2}\right)^2 = \frac{3n^2 + 4}{4}$, which has no real solutions.

If BM = -1, then $(n\ !\ 1)^2 \quad \frac{3n^2 \quad 4}{4}$ from which ! and = 8 yielding a triangle with sides of length 7, 8, and 9.

Case 3: The median is drawn to the longest side.

If AM = MB = CB, then $\frac{+}{} = \quad -$ from which = 2.

This gives a 2, 3, 4 triangle, which we have already considered.

If CM = MB = AM , then triangle ABC is a right triangle and the only right triangle with side lengths that are consecutive integers is the 3, 4, 5 triangle already considered.

The only other possibilities for AMC or AMB to be isosceles is if CM = or –1. Using the law of cosines on ABC,

$$(n\ !\ 1)^2 = n^2 + (n+1)^2\ !\ 2n(n+1)\cos A \text{ from which}$$

(7, 8, 9). The three triangles are shown below.