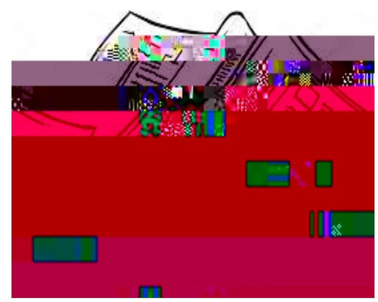
**THE 2022  2023 KENNESAW STATE UNIVERSITY**
**HIGH SCHOOL MATHEMATICS COMPETITION**

PART I – MULTIPLE CHOICE

*For each of the following 25 questions, carefully blacken the appropriate box on the answer sheet with a #2 pencil.  Do not*



6. A farmer bought some chicks and paid a total of $420. He paid the same amount for each chick. If each chick had cost a dollar more, he would have obtained 2 fewer chicks for the same amount of money. How many chicks did he buy?

(A) 20 (B) 24

14. Debbie and Don were comparing their stacks of pennies. Debbie said “If you gave me a

21. Define a set of positive integers to be *balanced* if the set is not empty and the number of even integers in the set is equal to the number of odd integers in the set. How many subsets of the set of the first 10 positive integers are balanced?

(A) 251 (B) 225 (C) 146 (D) 31 (E) None of these

22. Given square ABCD with side length 1. Fuare ABCD with side length 1. Fuare ABCD wit re A

## Solutions

1\. C Pages on the same side of a sheet add to 101. Therefore, the four pages are 15, 16, 85 and 86. The desired sum is 186.

2\. A Any even value of $k$ will make an even number, and 535555555 is divisible by 5. Both 3 and 9 make the number divisible by 3, and the divisibility test for 11 shows that 11 divides it when $k$ equals 7. The only value of $k$ left is 1, and since we are told that the number is prime for at least one value of $k$, $k$ must equal 1.

3\. D The product of the roots of $\;^2 + \;\; + \;\; = 0$ is $- = 6$ $\quad c = 6a$

The product of the roots of $\;^2 + \;\; + \;\; = 0$ is $- = 8$ $\quad = \frac{}{8}$

The product of the roots of $\;^2 + \;\; + \;\; = 0$ is $-$, and $- = \frac{\frac{}{8}}{6} = \frac{1}{48}$

[redacted]

15. C Note that since *S* contains the first 32 positive odd integers, the sum of its elements is $32^2 = 1024$. Thus, the problem is equivalent to finding the number of subsets of *S* whose elements each have a sum of 24. We can count these by hand: {23,1}, {21,3}, {19,5}, {17,7}, {15,9}, {15,5,3,1}, {13,11}, {13,7,3,1}, {11,7,5,1}, {11,9,3,1}, and {9,7,5,3}. Therefore, there are 11 such subsets.

16. E Label the diagram as shown and note that EF = 1, ED = 8,
PA' = 5, and FA = *h*. Construct $\overline{EA}$. Because PBA is a
right triangle, BA = 3, making DA = *h* – 3.
Using the Pythagorean Theorem on DEA, $(\ \ - 3)^2 + 8^2 = (EA\ )^2$
Using the Pythagorean Theorem on FEA, $\ ^2 + 1^2 = (EA\ )^2$.
Clearing parentheses, subtracting the two equations and solving for *h*
gives *h* = 12.

17. B Converting each equation into logarithmic form and using the change of base formula:

$$= \left(\frac{\log 4}{\log 3}\right)\left(\frac{\log 5}{\log 4}\right)\left(\frac{\log 6}{\log 5}\right)\left(\frac{\log 7}{\log 6}\right)\left(\frac{\log 8}{\log 7}\right)\left(\frac{\log 9}{\log 8}\right) = \frac{\log 9}{\log 3} = 2.$$

18. B <u>Method 1</u>: Let the terms of the arithmetic sequence be *a* – *d*, *a*, *a* + *d*. Thus, 3*a* = 96, and *a* = 32, making the terms of the arithmetic sequence 32 – *d*, 32, 32 + *d*. Let the terms of the geometric sequence be $-$, *b*, *br*. Therefore, 32 + *b* = 56, and *b* = 24, making the

$2 - 2x\sqrt{3} = 3x \qquad 2 = x(2\sqrt{3} + 3) \qquad x = \frac{2}{2\sqrt{3}+3} = \frac{4\sqrt{3}-6}{3}$

Therefore, BF = $\left(\frac{4\sqrt{3}-6}{3}\right)\sqrt{3} = \frac{12-6\sqrt{3}}{3} = 4 - 2\sqrt{3}.$

23. B Let’s examine the first few terms of the sequence. $\quad = \frac{1 + \quad _{-1}}{\quad _{-2}}$ when $\quad _1 = 20,$ and $\quad _2 = 22.$

$_1 = 20, \quad _2 = 22, \quad _3 = \frac{1+22}{20} = \frac{23}{20}, \quad _4 = \frac{1+\frac{23}{20}}{22} = \frac{43}{(20)(22)},$

$_5 = \frac{1+\frac{43}{(20)(22)}}{\frac{23}{20}} = \frac{483}{(22)(23)} = \frac{21}{22}, \quad _6 = \frac{1+\frac{21}{22}}{\frac{43}{(20)(22)}} = 20, \quad _7 = \frac{1+20}{\frac{21}{22}} = 22.$

Therefore, $_1 = \quad _6$ and $_2 = \quad _7$, and the sequence repeats every 5 terms.

Since 2022 $\quad 2 \bmod 5$, $_{2022} = \quad _2 = 22.$