THE 2023–2024 KENNESAW STATE UNIVERSITY

marked. Note that wild guessing is likely to lower your score. When the ex
your proctor. You may keep your copy of the questions.

NO CALCULATORS

1. A number is Beprisque if it is the only integer between a prime number and a perfect square.

13. One of the roots of the polynomial equation $x^4 + ax^2 + b = 0$ is $\sqrt{2} + \sqrt{5}$. If a and b are rational numbers, compute the value of b – a.

(A) 3 (B) 7 C) 21 (D) 23 (E) 29

14. Compute the value of $\left(\log_{2023}\frac{1}{2}\right)\left(\log_{2022}\frac{1}{3}\right)\left(\log_{2021}\frac{1}{4}\right)\cdots\left(\log_{3}\frac{1}{2022}\right)\left(\log_{2}\frac{1}{2023}\right)$

(A) –2023 (B) –1 (C) 1 (D) 2023 (E) None of these

15. If $c^2 = (a + bi)^2 - 130i$, where i is the imaginary unit and a, b, and c are positive integers, compute c.

(A) 9 (B) 10 (C) 11 (D) 12 (E) 13

16. The first two positive integers n for which $1 + 2 + 3 + \ldots + n$ is a perfect square are 1 and 8.

20. In the magic square shown, each row, column, and main diagonal sum to 100, where T, R, I, A, N, G, L, E

Solutions

1. D Listing theperfectsquares less than 100, it is easy to identify the Beprisque numbers as 2, 3, 8, 10, 24, 48, 80, 82, for a total of 8.

2. C There are 2 possibilities shown,

(N+ O)^6 = 2 NO+ 33. However, NO= ? and N+ O= –3. Thus, (–3)^6 = 2 ?+ 33 and ?= –12

10. C Let p = number of pennies, 2p = number of dimes, and 6p = number of quarters. Then the value of the coins in cents is p + 20p + 150p = 171p. The amount of money in the bag must be divisible by 171. Only choice (C) works.

11. B Let U= B$^{?5}$(T). Then, T= $\frac{ì > 5}{ì ? 5}$

17. B Let x equal the area of ΔABO. Since the area of ΔBCA is 2, it follows that the area of ΔBCO is equal to 2 ī x. Similarly, since ΔABD has area 1, we see that ΔDAO has area equal to 1 ī x. Finally, since ΔDAC has area 3, we conclude that the area of ΔCDO is equal to 3 ī (1 ī x) = 2 + x. In particular, the area of ΔCDB is (2 ī x) + (2 + x) = 4. Now ΔABO and ΔBCO share the same altitude to AC, so their areas are proportional to the lengths of their bases, namely AO and OC. Similarly, the areas of ΔDAO and ΔCDO are also proportional to AO and OC. Thus $\frac{E S}{S G} = \frac{ë}{6 ? ë} = \frac{5 ? ë}{6 > ë}$ ⇒ 2x + T^6 = T^6 ī 3x + 2, from which x = area of ΔABO = $\frac{6}{9}$.

18. A Since a_1, a_2, a_3 are in arithmetic sequence, $=_7$ F $=_6$ = $=_6$ F1 and $=_7$ = 2 $=_6$ F1. Therefore, $=_6$, 2 $=_6$ F1, $=_8$ are in geometric sequence which gives us or $=_8$ = $\frac{(6 Ọ ? 5)^{·}}{Ọ}$. Now, 2 $=_6$ F1, $\frac{(6 Ọ ? 5)^{·}}{Ọ}$, $=_9$ are in harmonic sequence, which gives us $\frac{6 Ọ ? 5}{(6 Ọ ? 5)^{·}}$, $\frac{Ọ}{(6 Ọ ? 5)^{·}}$, $\frac{5}{Ọ_1}$ are in arithmetic sequence. Therefore, $\frac{5}{Ọ_1} = \frac{5}{(6 Ọ ? 5)^{·}}$ Hence $=_9$ = 5 = (2 $=_6$ F1)^6 = ($=_7$)^6. Thus, $=_7$ = ³⁄₅.

19. E =

21. D Let the coordinates of the other vertex of the longer leg ,be = 6 ( ). = Then

Noting that $\cos^6 15\ \mathrm{F} \sin^6 15 = \cos 30 = \frac{\sqrt{3}}{2}$ and $\cos^6 15 + \sin^6 15 = 1$, this last equation becomes $3T^6\ \mathrm{F}\, 4\sqrt[3]{3}\, T + 3 = 0$